

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2

Branch: B. Tech (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) The infinite series $1 + r + r^2 + \dots + r^{n-1}$ is convergent if
 (A) $|r| < 1$ (B) $|r| > 1$ (C) $r = 1$ (D) $r < -1$
- b) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (A) $\log 2$ (B) zero (C) infinite (D) none of these
- c) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then $(f_n + f_{n-2})$ is equal to ?
 (A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$
- d) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x \, dx$ is
 (A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$
- e) $\sqrt{\frac{1}{2}} \sqrt{\frac{3}{2}} \sqrt{\frac{5}{2}} = \text{_____}$
 (A) $\frac{3}{8}(\pi)^{\frac{3}{2}}$ (B) $\frac{3}{8}(\pi)^{\frac{5}{2}}$ (C) $\frac{3}{8}(\pi)^{\frac{1}{2}}$ (D) $\frac{1}{8}(\pi)^{\frac{3}{2}}$
- f) Duplication formula: $\sqrt{n} \sqrt{n + \frac{1}{2}} = \text{_____}$
 (A) $\frac{\sqrt{\pi} \sqrt{n}}{2^{2n-1}}$ (B) $\frac{\sqrt{\pi} \sqrt{2n}}{2^{n-1}}$ (C) $\frac{\sqrt{\pi} \sqrt{2n}}{2^{2n-1}}$ (D) $\frac{\sqrt{\pi} \sqrt{n}}{2^{n-1}}$
- g) $erf(x) + erf_c(x)$ is equal to
 (A) 0 (B) 1 (C) -1 (D) 2



- h) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-2\sin^2\theta}}$ is equal to
 (A) $\frac{1}{\sqrt{2}}E\left(\frac{1}{\sqrt{2}}\right)$ (B) $\frac{1}{2}K\left(\frac{1}{\sqrt{2}}\right)$ (C) $\frac{1}{\sqrt{2}}K\left(\frac{1}{\sqrt{2}}\right)$ (D) $\frac{1}{2}E\left(\frac{1}{\sqrt{2}}\right)$
- i) The tangents at the origin are obtained by equating to zero
 (A) the lowest degree terms (B) the highest degree terms
 (C) constant term (D) none of these
- j) If the powers of x are even, then the curve is symmetrical about
 (A) X – axis (B) Y – axis (C) about both X and Y axes (D) None of these
- k) $\int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-r^2} \cdot r \, dr \, d\theta$ is equal to
 (A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$
- l) The transformations $x + y = u$, $x - y = v$ transform the area element $dy \, dx$ into $|J| \, du \, dv$, where $|J|$ is equal to
 (A) $\frac{1}{2}$ (B) 1 (C) u (D) none of these
- m) The degree and order of the differential equation of all parabolas whose axis is x-axis are
 (A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these
- n) Solution of differential equation $x \, dy - y \, dx = 0$ represents
 (A) Rectangular hyperbola (B) Circle whose centre is at origin
 (C) Parabola whose vertex is at origin
 (D) Straight line passing through origin

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Using reduction formula prove that $\int_0^a x^5 (2a^2 - x^2)^{-3} \, dx = \frac{1}{2} \left(\log 2 - \frac{1}{2} \right)$. (5)

b) Prove that $\int_0^{\infty} \frac{x^4}{4^x} \, dx = \frac{24}{(\log 4)^5}$. (5)

c) Evaluate: $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) \, dz \, dy \, dx$ (4)

Q-3 Attempt all questions (14)

a) Prove that $\int_0^1 x^5 (1-x^3)^{10} \, dx = \frac{1}{3} B(2,11)$. (5)

b) Solve: $\frac{dy}{dx} + 2y \tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$ (5)



c) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$. (4)

Q-4 Attempt all questions (14)

a) By changing into polar co-ordinates, evaluate the integral (5)

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dx dy .$$

b) Examine the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2+1} + \dots$ for convergence using ratio test. (5)

c) Using reduction formula evaluate: $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$ (4)

Q-5 Attempt all questions (14)

a) Solve: $\frac{(x-2y) dy}{(3x+y) dx} = 3x^2 - 5xy - 2y^2$ (5)

b) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dy dx$ and hence evaluate it. (5)

c) Prove that $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx = \frac{1}{5005}$. (4)

Q-6 Attempt all questions (14)

a) Examine the series $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$ for convergence using root test. (5)

b) Using reduction formula prove that $\int_0^{\pi} x \cos^6 x dx = \frac{5\pi^2}{32}$. (5)

c) Solve: $(x^2 + y^2 + 1)dx - 2xy dy = 0$ (4)

Q-7 Attempt all questions (14)

a) Trace the curve $y^2(2a-x) = x^3$. (5)

b) Find the area enclosed by the cardioid $r = a(1 - \cos \theta)$. (5)

c) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}}$ (4)

Q-8 Attempt all questions (14)

a) For small values of x, show that $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + \dots \right)$. (5)

b) Trace the curve $r = a(1 + \cos \theta)$. (5)

c) Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ (4)

